A KINETIC VARIANT OF THE THEORY OF CREEP AND LONGTERM STRENGTH OF METALS

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The most well-grounded hypothesis for description of creep in metals is Rabotnov's theory of kinetic equations with structural parameters [1]. In the absence of any strengthening effect the simplest case is that of similarity of the creep diagrams in different directions, where $p(t) = \Phi(\sigma)M(t/t_f)$, with p being the creep deformation; t, the current time; σ, stress; t_f, time to failure. To describe such a creep process (second and third stages) only one structural parameter is needed. There exist three well-tested phenomenological relationships [2, 3], [4-6], [7, 8]:

$$\dot{p} = C\sigma^m \exp(\alpha p),\ p(0) = 0, \tag{1}$$

where C, m, α are constants of the material and the dot above indicates differentiation with respect to time, temperature being assumed constant;

$$\dot{A} = C\sigma^m/(A_* - A)^\alpha,\ A(0) = 0, \tag{2}$$

where $A(t) = \int_0^t \sigma dp$ is the specific scattering energy of the creep process; A_* is a constant of the material;

$$\dot{p} = C\sigma^m p^\alpha,\ p(0) = 0\ (0 < \alpha < 1). \tag{3}$$

Equation (3) is based on Grekhem's exponential law of accelerating creep ($p \sim t^k$, $k > 1$), which is found in nickel alloys (ÉI 437B (see Eq. (22)), ÉI 617, ÉI 867, Nimonic, and others [7, 8]) at a high stress level (before the breakpoint in longterm strength curves).

In the general case creep curves show a hardening stage. In [9, 10], following Andrade's hypothesis [4] the total creep deformation p_Σ was represented as the sum of the component now under consideration and the first stage deformation p_I: $p_\Sigma = p_I + p$. For constant stresses the creep process in pure metals and some alloys (Nimonic 80A, chromium–nickel austenic steels, etc.) shows two deformation components with the second satisfying Davis's exponential disordering law

$$p = K \exp(\lambda t), \tag{4}$$

where K, λ are independent of time. Each of Eqs. (1)-(4) is applicable only over a quite narrow range of temperature and stress. The question of possible intersection of their applicability ranges is important, since it is not clear which of the models is to be preferred. Moreover (as will be shown below), the models of Eqs. (1)-(4) do not permit coverage of the temperature-force range in which modern constructions must operate. The goal of the present study is the creation of a corresponding generalized model. The single-parameter model to be developed contains Eqs. (1)-(4) as special cases and has a wider range of applicability than Eqs. (1)-(4); it follows from the model that the ranges of applicability of Eqs. (1)-(4) do not overlap.

The structural parameter of the basic equations should not involve the mechanical parameters (p, A), but rather physical quantities characterizing the state of the material's microstructure [11]. In the general case, given similarity of the creep diagrams, the kinetic equations will have the form [1]

$$\dot{p} = f(\sigma)\Psi(a),\ \dot{a} = \varphi(\sigma)G(a), \tag{5}$$

where a is a structural parameter. The stress dependence can usually be chosen as power-like [1] (see Eqs. (1)-(3)).

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The choice of a function to describe dependence on the parameter a remains open. To develop a phenomenological theory of creep knowledge of the physical meaning of the structural parameter is not obligatory. Since the experimental parameter a is not defined, models can be considered equivalent if the creep deformation calculated by them is identical and the equations are defined to the accuracy of an arbitrary (nondegenerate) change of the variable a (a = H(x)). Let there exist a converse function Ψ^{-1}, then for Ψ we may choose an arbitrary (sufficiently smooth) function (transformation from Ψ to the arbitrary Ψ_* is accomplished by the replacement of variables $a = \Psi^{-1}[\Psi_*(x)]$). We take $\Psi(a) = \exp(\kappa a)$. Then consideration of any of Eqs. (1)-(4) leads to a function G of the same form ($G(a) = \exp(\kappa a)$) and we write Eq. (5) as

$$\dot{p} = B\sigma^n \exp(qa),\ p(0) = 0,\ \dot{a} = L\sigma^m \exp(\kappa a),\ a(0) = a_0, \tag{6}$$

where B, n, q, L, m, κ, a_0 are constants of the material.

The presence of a power dependence on creep deformation in Eq. (1) can be interpreted (according to the theory of thermodynamic fluctuations) [2, 3] as a linear dependence of activation energy on the quantity $p(\alpha = \alpha_0/T$, T is the absolute temperature, α_0 is a constant of the material). A similar explanation can be offered for the origin of the exponential functions in the model of Eq. (6) – linear dependence of the activation energy of the parameter a ($q = q_0/T$, $\kappa = \kappa_0/T$, q_0, κ_0 are constants of the material), which can then be interpreted as the characteristic dimension of some developing microdefects.

Without limiting generality, we take $q \geq 0$, so that the case $q < 0$ reduces to a replacement $a = -x$. Then from the condition $\dot{p} \geq 0$ and $\ddot{p} > 0$ it follows that $B \geq 0$ and $L > 0$. The linear replacement $a = c_1 x + c_2$, containing two arbitrary parameters c_1, c_2 does not change the form of the equations. Therefore two constants of system (6) (for example L, κ) can be chosen arbitrarily. We will consider special cases of the model of Eq. (6).

1. $q = \kappa$, so that $a = a_0 + (L/B)z\left(z = \int_0^t \sigma^{-\gamma} dp,\ \gamma = n - m\right)$ and the model of Eq. (6) can be written in the form

$$\dot{z} = B\sigma^m \exp\{q(a_0 + (L/B)z)\}. \tag{7}$$

It is evident that at $\gamma = 0$ ($z \equiv p$) the models of Eqs. (1) and (7) are equivalent.

Let $\kappa \neq q$, $q \neq 0$, whereupon it is convenient to reduce the model of Eq. (6) to a simpler ("canonical") form by a replacement of the variable $l = B/[L(q - \kappa)] \exp\{a(q - \kappa)\}$:

$$\dot{p} = D\sigma^n l^r,\ p(0) = 0,\ \dot{l} = \operatorname{sign}(r) D\sigma^m l^r,\ l(0) = l_0, \tag{8}$$

where

$$r = q/(q - \kappa);\ D = B\left[\operatorname{sign}(r)\frac{Lq}{Br}\right]^r;$$

$$l_0 = \operatorname{sign}(r)\frac{Br}{Lq}\exp\left(\frac{q}{r}a_0\right);\ \operatorname{sign}(r) = \begin{cases} 1, r > 0, \\ -1, r < 0. \end{cases}$$

For the model of Eq. (6) $q = 0$ is equivalent to $r = 0$ for the model of Eq. (8).

2. $q < \kappa (r < 0)$, so that $l = l_0 - z$ and the model of Eq. (8) will be

$$\dot{z} = D\sigma^m/(l_0 - z)^{|r|}. \tag{9}$$

At $\gamma = -1$ ($z \equiv A$) the models of Eqs. (2), (9) coincide to the accuracy of the notation used.

3. $\kappa < 0$ ($0 \leq r < 1$). Then

$$\dot{z} = D\sigma^m(l_0 + z)^r. \tag{10}$$

When $l_0 = \gamma = 0$, the second stage of creep is absent and the model of Eq. (10) becomes equivalent to that of Eq. (3). For $l_0 \neq 0$ the creep curves have a steady-state stage, as found under short-term creep conditions in austenittic steels (see Eq. (20)), magnesium, and other heat-resistant alloys [12].

4. $\kappa = 0$ ($r = 1$).

5. $q > \kappa > 0$ ($r > 1$).

These cases reduce to the solution of Eq. (10), but differ in principle from the previous case ($\kappa < 0$) in that at $l_0 = 0$ Eq. (10), like Eq. (3), has only a trivial solution: $z(t) \equiv 0$.

Depending on the material in question, any of five cases of the model of Eq. (6) can be realized. In this case type 1 (Fig. 1 for titanium alloy), 2 (Eq. (20) for stainless steel), 4 (Table 1 for α-iron) and 5 (Eqs. (19), (22) for aluminum and nickel alloys) creep processes are usually observed in lengthier tests (hundreds and thousands of hr).

For a description of longterm strength Eq. (6) must be supplemented by the final condition $a(t_f)$ where t_f is the time to failure): following the hypothesis of [1], we take $a(t_f) = a_* = \text{const}$. We then have a power law for longterm strength $\log t_f = R - m \log \sigma$ (where R is a constant of the material) and the model of Eq. (6) satisfies the laws of linear summation of degradations and the quantities z, which have been confirmed experimentally under conditions of similarity of creep diagrams [4]. Moreover, the condition referred to corresponds to the criteria $p(t_f)$ and $A(t_f) = \text{const}$, by which Eqs. (1)-(3) are usually supplemented.

In the general case, it is necessary to define six constants in Eq. (6) (n, m, D, r, l_0, $l_* = l(t_f)$). Two of these are defined from the longterm strength curve, and four from creep curves. At constant stresses, integrating Eq. (8), we obtain

$$\left.\begin{aligned} p &= \operatorname{sign}(r)\,\sigma^{\nu}(l - l_0), \\ l &= l_0\left\{1 - \left[1 - (l_*/l_0)^{1-r}\right]\frac{t}{t_p}\right\}^{1/(1-r)}, \\ t_f &= \left(l_0^{1-r} - l_*^{1-r}\right)\left[(r-1)\operatorname{sign}(r)\,D\sigma^m\right]^{-1} \end{aligned}\right\} r \neq 1, \tag{11}$$

$$\left.\begin{aligned} l &= l_0 \exp\left(D\sigma^m t\right), \\ t_f &= \ln\left(l_*/l_0\right)(D\sigma^m)^{-1} \end{aligned}\right\} r = 1.$$

At $q = \kappa$ in Eq. (6) it is convenient to perform the replacement $a = xL/B + a_0$, then

$$\dot{z} = \dot{x} = C\sigma^m \exp \alpha x, \; z(0) = x(0) = 0. \tag{12}$$

Here $C = B \exp(qa_0)$; $\alpha = qL/B$. For constant stresses, integrating Eq. (12), we have

$$p = -\frac{\sigma^{\nu}}{\alpha}\ln\left\{1 - [1 - \exp(-\alpha x_*)]\,t/t_f\right\}, \tag{13}$$

$$t_f = [1 - \exp(-\alpha x_*)](\alpha C\sigma^m)^{-1},$$

where $x_* = x(t_f)$.

Thus, with consideration of Eqs. (11), (13), creep deformation can be written in the form

$$r = \infty\,(q = \varkappa):\; p = l_0'\sigma^{\nu}\ln\left(1 - c\frac{t}{t_f}\right)^{-1}, \tag{14}$$

$$l_0' = 1/\alpha, \quad c = 1 - \exp(-\alpha x_*);$$

$$r < 0\,(q < \varkappa):\; p = l_0\sigma^{\nu}\left[1 - \left(1 - c\frac{t}{t_f}\right)^{\beta}\right], \tag{15}$$

$$c = 1 - (l_*/l_0)^{1-r}, \quad \beta = 1/(1-r);$$

$$0 \leqslant r < 1\,(\varkappa < 0):\; p = l_0\sigma^{\nu}\left[\left(1 + c\frac{t}{t_f}\right) - 1\right], \tag{16}$$

$$c = (l_*/l_0)^{1-r} - 1, \quad \beta = 1/(1-r);$$

$$r = 1\,(\varkappa = 0):\; p = l_0\sigma^{\nu}\left[\exp\left(c\frac{t}{t_f}\right) - 1\right], \tag{17}$$

$$c = \ln(l_*/l_0);$$

TABLE 1

σ, kg/mm^2	t_f, h	λ, h^{-1}	12,17/t_f, h^{-1}	$K\cdot10^{-8}$	$6{,}27\cdot10^{-12}\,\sigma^{2{,}6}$
70,31	1,97	6,42	6,18	40,14	39,7
62,4	5,57	2,06	2,19	29,21	29,16
56,25	25	0,48	0,487	22	22,26
38,67	350	0,036	0,0348	8,43	8,4

TABLE 2

t/t_f	$1-(1-0{,}9946\frac{t}{t_f})^{0,3}$	$1-(1-t/t_f)^{0,3}$
0,95	0,58	0,59
0,98	0,67	0,69
0,995	0,75	0,8
1	0,79	1

TABLE 3

T,°C	l_0', kg/mm^2
450	1,86
500	1,92
550	1,37

$$r>1\ (q>\varkappa>0):\quad p=l_0\sigma^{\gamma}\left[\left(1-c\frac{t}{t_f}\right)^{-\beta}-1\right],\qquad c=1-(l_0/l_*)^{r-1},\quad \beta=1/(r-1). \tag{18}$$

In Eq. (17) the constant c is quite large (c > 10 [9, 10]). Moreover, in the vicinity of t = 0 usually p << p_I, therefore Eqs. (4) and (17) prove to be practically identical. Limitations are introduced only by the approximation of the coefficients of Eq. (4) $\lambda = c/t_f$, $K = l_0\sigma^{\gamma}$, which follows from Eq. (17). The possibility of such an approximation is demonstrated by Table 1, which shows values of the coefficients λ, K for creep in α-iron (Fe_α) at a temperature of 542°C [9] and their approximation by the relationships referred to (γ = 2.6).

Thus, to obtain the model of Eq. (6) it is sufficient to require the correspondence of Eq. (5) to one of the four theories of Eqs. (1)-(4). In this case Eq. (6) "automatically" contains the other theoreies as well. This is a convincing argument in favor of the validity of the kinetic concept proposed.

The constants $\gamma, l_0(l_0')$, c, β of Eqs. (14)-(18) can be found in the following manner. The value of γ can be calculated from the similarity conditions for the curves $p(t)\sigma^{-\gamma}$ in different directions (in particular, from the condition $p(t_f)\sigma^{-\gamma}$ = const). The constant $l_0(l_0')$ can be eliminated from the expressions by considering the relationship of the current deformation p(t) to its value at some fixed point in time. In the general case the constants c and β can be determined numerically.

However in Eq. (15) outside the vicinity of $t = t_f$ (Table 2) one can usually use the approximation $l_* \to \infty$, i.e., c = 1 (which corresponds to Eq. (2)); c = 1 ($x_* \to \infty$) can also be used in Eq. (14) (see Fig. 1). For Eq. (16) possible simplifications are $l_0 \to 0$ (see Eq. (22)) outside the vicinity of t = 0 (which corresponds to Eq. (3)) or $ct/t_f << 1$ outside the vicinity of $t = t_f$ (which is equivalent to steady-state flow theory for short term creep [12]). In the cases mentioned it is possible to separate the definitions of the variables c and β. The range of the variable r (Eqs. (14)-(18)) is chosen for best agreement with experiment. With correct selection of Eqs. (15), (16), or (18) the calculated value of r usually lies within the interval $(-\infty, 0)$, (0, 1), $(1, \infty)$, while with incorrect selection it takes on a value close (or tending) to the boundary of the interval ($r \approx -\infty$, 0, 1, $+\infty$). This fact allows simplification of the calculation case.

Figures 1 and 2 show experimental data on the creep of the titanium alloy OT-4 and the alloy D16T [5]. The experimental results are shown by solid lines (in all graphs). The tests were performed at constant stresses (with compensation for the change in area of the specimen cross section). As follows from the calculations, the creep fo the alloy OT-4 is described well at any value of r satifying the condition $|r| > 10$, so that the value of r can be assumed temperature-independent (in the general case, r = f(T) [6]). This indicates that the constant r is superfluous, and a type 1 process is realized 1 (r = ∞). The

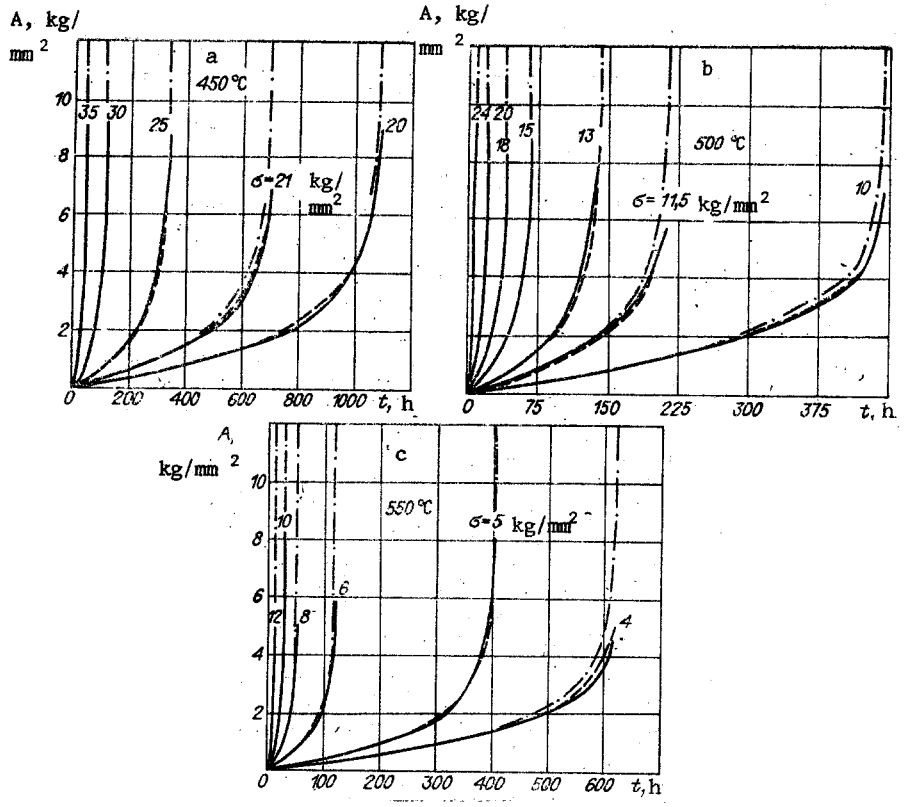

Fig. 1

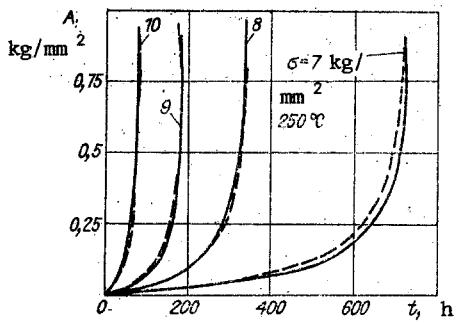

Fig. 2

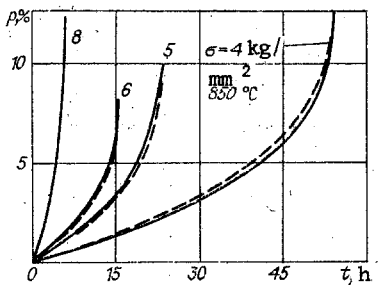

Fig. 3

calculated creep diagrams for the alloy OT-4 were constructed with Eq. (14) (see Table 3 for values of constant $\ell_0^!$): c = f(A_*), dashed lines; c = 1, dash-dot lines.

The creep characteristics of the alloy D16T are described best by Eq. (18) with r > 1 (at r > 1 the error of the approximation comprises 5-10%, while for r < 0 and r = ∞ it is of the order of 30%). Calculated values for D16T (see Fig. 2, dashed lines) are described by the equation (r = 3)

$$A = 0{,}17[(1 - 0{,}9753t/t_f)^{-0{,}5} - 1]. \qquad (19)$$

The dimensions of the numerical values of the constants presented in the expressions correspond to the dimensions of A, σ, p, t indicated on the graphs.

The reason for the absence of similarity of the stress diagrams may be the presence of a first stage ($p_I \neq 0$) [1, 9, 10], or the realization of a mixed failure process [13]. Characteristic examples of the latter are shown in Fig. 3 [14] (Kh18N10T steel) and Fig. 4 [7] (high temperature nickel alloy ÉP437B). On the longterm strength curve the point of transition from one form of failure to the other is "angular" [13] (Fig. 4). In the description of longterm strength the classical hypothesis of independence of the various failure processes [1] was used. In this hypothesis $t_f = \min_i t_{fi}$ (where i enumerates the types of failure). The thermodynamic fluctuation theory can serve as a basic for accepting the same hypothesis [15] for the corresponding creep processes, than $p = \Sigma p_i$. Each component of p_i is described independently by Eqs. (6)-(18). For approximate determination of the

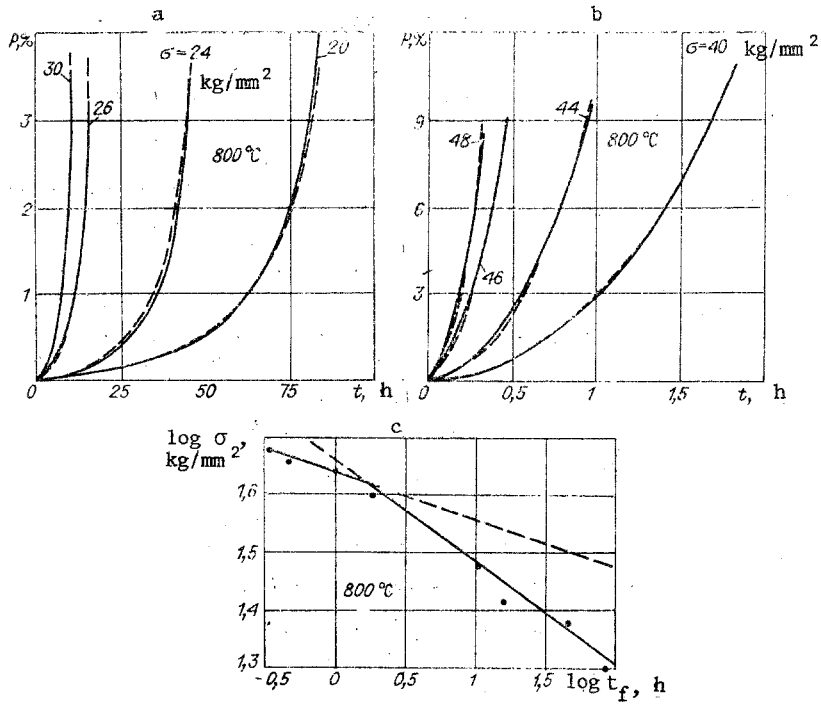

Fig. 4

TABLE 4

σ, kg/mm²	t_f, h	t_{f1}, h	t_{f2}, h	σ, kg/mm²	t_f, h	t_{f1}, h	t_{f2}, h
48	0,328	0,328	0,784	30	10,3	73,5	10,3
46	0,462	0,462	0,922	26	15,4	173	15,4
44	0,975	0,975	1,32	24	45,9	1730	45,9
40	1,82	1,93	1,82	20	83,7	6120	83,7

model constants γ_i, ℓ_{0i}, c_i, β_i the method described above was used, but in the stress range where the effect of other components p_j can be neglected, $j \neq 1$. The values of the constants found can be refined by the successive approximation method.

The longterm strength curve of Kh18N10T can be described satisfactorily by a power law [16], therefore it was assumed that $p = p_1 + p_2$, $t_{f2} = t_f$. Calculations showed that the component p_1 is significant only at $\sigma = 8$ kg/mm² (see Fig. 3) and has the form of Eq. (16) ($r = 2/3$). The constants of the component p_2, Eq. (15) ($r = -2, 3$) were determined from the creep curves for $\sigma = 4, 5, 6$ kg/mm². The expressions obtained were:

$$p_1 = \begin{cases} 0, & \sigma \leqslant 6 \text{ kg/mm}^2, \\ 0{,}034\left[(1 + 1{,}17t)^{2{,}5} - 1\right], & \sigma = 8 \text{ kg/mm}^2, \end{cases} \qquad (20)$$

$$p_2 = 15{,}8\frac{4}{\sigma}\left[1 - (1 - 0{,}9946t/t_{f2})^{0{,}3}\right].$$

With a power approximation of the quantity t_{f1} to determine the constants at least two creep curves with the comopnent p_1 nonzero are needed. The experimental data consisted were insufficient to calculate p_1 at $\sigma \neq 8$ kg/mm². As is evident from Fig. 3, the dashed lines constructed with Eq. (20) describe the experimental curves satisfactorily. It follows from the data of Table 2 that everywhere but at the point $t = t_f$ in Eq. (15) we can take $c = 1(\ell(t_f) \to \infty)$.

The longterm strength curve for ÉI 437B (Fig. 4) appear as straight lines with a bend point (σ_b). To calculate creep curves with Eqs. (14)-(18) it is necessary to extend these lines into the region indicated by the dashed lines, which leads to errors in determination of the values t_{f1} and t_{f2}, p_1 and p (especially in the vicinity of the bending point),

related to scattering of the experimental results, the influence of which can be reduced by using the hypothesis that the quantity t_f is a function of some "true" stress (σ_i), which is a random quantity [17].

With consideration of this

$$\lg t_{fi} = \begin{cases} \lg t_f, & i = i_*, \\ R_i - m_i \lg \sigma_t, & i \neq i_*; \end{cases} \quad \lg \sigma_t = \min_i \frac{R_i - \lg t_f}{m_i}, \tag{21}$$

where i_* is a number denoting the type of failure realized.

For the data considered the following constants were found: $R_1 = 19.7$, $m_1 = 12$, $R_2 = 9.5$, $m_2 = 5.7$ ($\sigma_b = 41.8$ kg/mm^2). Values of the quantities t_{f1} and t_{f2} calculated with Eq. (21) are presented in Table 4. Calculations showed that the component p_1 has the form of Eq. (16) ($r = 1/2$) with $\gamma = 0$ and proves to be insignificant at $\sigma \leq 30$ kg/mm^2. Therefore the initial approximation for the constants of the component p_2, Eq. (18) ($r = 1.7$) with $\gamma = 0$, were determined from creep curves at $\sigma = 20$, 24, 26, 30 kg/mm^2. The expression

$$p = 8{,}62 \left(\frac{t}{t_{f1}}\right)^2 + 0{,}335 \left[\left(1 - 0{,}806 \frac{t}{t_{f2}}\right)^{-1{,}5} - 1\right] \tag{22}$$

was obtained. It is evident from Fig. 4 that the dashed lines constructed with Eq. (22) describe the experimental results satisfactorily.

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